Analyzing demand

Intermediate Micro

Lecture 6

Chapter 6 of Varian
Analyzing demand

Can model utility function and decisions

- Even for $p, m$ we don’t observe
- Can use demand functions to model comparative statics
  - Comparative statics: Studying the effect on the equilibrium outcome due to a change in parameters.
  - How does demand change as income increases?
  - What are the effects on demand of a change in prices?
  - Categorize goods based on comparative statics
Review: Demand function

Start with basic consumer decision problem:

\[
\max_{x_1, x_2} u(x_1, x_2) \\
\text{s.t. } p_1 x_1 + p_2 x_2 = m
\]

Leave \( p_1, p_2, m \) as parameters, and obtain demand functions

\[
x_1(p_1, p_2, m) \\
x_2(p_1, p_2, m)
\]
Changing $m$

\[ x_1(p_1, p_2, m) \]

- Consider effect of $\uparrow m$
- Easiest measure: \( \frac{dx_1}{dm} \) (\( = \frac{d}{dm} x_1(p_1, p_2, m) \))
- **Normal good**: \( x_1 \) is a normal good (at \((p_1, p_2, m))\) if \( \frac{dx_1}{dm} > 0 \)
- **Inferior good**: \( x_1 \) is an inferior good (at \((p_1, p_2, m))\) if \( \frac{dx_1}{dm} < 0 \)
Changing Elasticity Examples Changing own price Changing other good’s price Examples

Normal vs Inferior

$x_1$ is normal
$x_2$ is normal

$x_1$ is inferior
$x_2$ is normal

Can both goods be inferior?
Income offer curve

- **Income offer curve**: A graph of all optimal bundles for a given \( p_1, p_2 \), for all values of \( m \)
- \( m \) varies
- \( p_1, p_2 \) stay constant
- Plug various values of \( m \) into demand functions, plot results
- If both goods are normal, income offer curve is upward sloping (↗)
Income offer curve

- **Income offer curve**: A graph of all optimal bundles for a given $p_1, p_2$, for all values of $m$
- $m$ varies
- $p_1, p_2$ stay constant
- Plug various values of $m$ into demand functions, plot results
- If both goods are normal, income offer curve is upward sloping (↗)
Engel curve

- Engel curve: A graph of the demand for one of the goods, for all values of $m$, holding constant $p_1, p_2$
- $m$ varies
- $p_1, p_2$ stay constant
- Plug various values of $m$ into demand function, plot results
- If the good is normal, Engel curve is upward sloping (↗)
- The Engel curve never slopes downward
Changing $m$

Elasticity

Examples

Changing own price

Changing other good’s price

Examples

$\uparrow$

Income offer curve

axes: $x_1, x_2$

$\leftarrow$

Engel curves

axes: $x_i, m$
Elasticity - Not in book!

- **Income elasticity of demand:** \( \epsilon_{x_i,m} = \frac{dx_i}{dm} \times \frac{m}{x_i} \)

- This formula is called point (income) elasticity

- Percent change in \( x_i \) relative to the percent change in \( m \)
  - Non-calculus formula: \( \frac{\Delta x/x}{\Delta m/m} \)
  - Called arc (income) elasticity

- Formula for (instantaneous) percent growth of \( y \) due to \( z \):
  \( \frac{d}{dz} \ln(y(z)) \)

- \( \epsilon_{x_i,m} = \frac{\frac{d}{dm} \ln(x_1(p_1,p_2,m))}{\frac{d}{dm} \ln(m)} \)
Why use elasticity - Not in book!

\[ \frac{dx_i}{dm} : \text{change in } x_i \text{ due to increase in } m \]

- \[ \frac{dx_i}{dm} > 0: \text{increasing in } m \]
- \[ \frac{dx_i}{dm} < 0: \text{decreasing in } m \]
- Scale?

\[ \frac{dx_i}{dm} * \frac{m}{x_i} \text{ (income elasticity of demand): Same sign as } \frac{dx_i}{dm} \]

- Note that elasticity (and slope!) can vary with \( m \)
Elasticity-based definitions—Not in book!

- Unit elasticity: when 
  \[ \epsilon_{x_i,m} = 1 \]
- \( x_i \) grows at same rate as \( m \)
- Any ray **through the origin** has unit elasticity

Engel curve with unit elasticity
Homothetic preferences

- Homothetic preferences: A set of preferences with the property that, if \((x_1, x_2) \sim (y_1, y_2)\), then \((tx_1, tx_2) \sim (ty_1, ty_2), \forall t \geq 0\)

- Equivalent properties:
  - Income offer curves are straight lines through the origin, for any \((p_1, p_2)\)
  - Engel curves are straight lines through the origin, for any \((p_1, p_2)\)
  - \(\epsilon_{x_i, m}\) for any \((p_1, p_2, m)\), for any good \(i\)
Elasticity-based definitions- Not in book!

- Luxury good: $x_i$ for which $\epsilon_{x_i,m} > 1$
- $x_i$ grows at faster rate than $m$
- To identify on Engel curve
  1. Draw ray from origin to point
  2. If curve crosses ray from left to right, good is luxury at this $(p_1, p_2, m)$

Engel curve for Ikea furniture
Elasticity-based definitions- Not in book!

- Necessary good: $x_i$ for which $\epsilon_{x_i,m} < 1$
- $x_i$ grows at slower rate than $m$
- To identify on Engel curve
  1. Draw ray from origin to point
  2. If curve crosses ray from right to left, good is necessary at this $(p_1, p_2, m)$

Engel curve for Ikea furniture
Perfect substitutes

\[ u(x_1, x_2) = 2x_1 + 3x_2 \]
\[ m = x_1 + 2x_2 \]
\[ x_1(1, 2, m) = m, \ x_2(1, 2, m) = 0 \]
Cobb Douglas

\[ u(x_1, x_2) = x_1^{0.4} x_2^{0.6} \]
\[ m = 0.5x_1 + 1.5x_2 \]
\[ x_1(0.5, 1.5, m) = 0.8m, x_2(0.5, 1.5, m) = 0.4m \]

Income offer curve

Engel curve for \( x_2 \)
Quasilinear

\[ u(x_1, x_2) = \ln(x_1) + 0.25x_2 \]

\[ m = x_1 + x_2 \]

\[ x_1(1, 1, m) = \begin{cases} 
  m & \text{if } m < 4 \\
  4 & \text{if } m \geq 4 
\end{cases}, \quad 
 x_2(1, 1, m) = \begin{cases} 
  0 & \text{if } m < 4 \\
  m - 4 & \text{if } m \geq 4 
\end{cases} \]
Changing $p_1$: effect on $x_1$

$x_1(p_1, p_2, m)$

- Consider effect of ↑ $p_i$ on $x_i$
- Derivative: $\frac{dx_i}{dp_i} < 0$ for all known goods
- Giffin good: A good for which $\frac{dx_i}{dp_i} > 0$  
  - No documented examples!
**Own-price elasticity - Not in book!**

- **Own-price elasticity of demand:** $\epsilon_{x_i,p_i} = -\frac{dx_i}{dp_i} \cdot \frac{p_i}{x_i}$
- %↓ in $x_i$ relative to %↑ in $p_1$

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<td>$\uparrow p_i \Rightarrow x_i = 0$, $\downarrow p_i \Rightarrow x_i = \infty$</td>
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Inverse demand

\[ x_1(p_1, p_2, m) \]

- Take \( m, p_2 \) as fixed
- Rewrite demand function as \( x_1(p_1) \)
- Can find inverse demand function: \( p_1(x_1) \)
  - Gives \( p_1 \) that causes \( x_1 \) to be optimal
  - Only exists if each value \( x_1 \) optimal only for one \( p_1 \)
Implications of own-price elasticity - Not in book!

- Expenditure on good 1 = $p_1 x_1$
- If $\epsilon_{x_i,p_i} > (\leq, <) 1$
  - $\uparrow p_1$ causes $\downarrow$ (no change, $\uparrow$) in $p_1 x_1$

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Changing $p_1$: effect on $x_2$ - Not in book!

\[ x_2 = \frac{m}{p_2} - \frac{p_1 x_1}{p_2} \]

- Suppose $\uparrow p_1$
- $\frac{dx_2}{dp_1} > (=, <) 0$ when $\epsilon_{x_i, p_i} > (=, <) 1$
- Complements: Two goods for which $\frac{dx_2}{dp_1} < 0$
- Substitutes: Two goods for which $\frac{dx_2}{dp_1} > 0$
**Price offer curve**

- **Price offer curve**: A graph of all optimal bundles for a given \( m, p_2 \), for all values of \( p_1 \)
- \( p_1 \) varies, \( m, p_2 \) constant
- Plug values of \( p_1 \) into demand functions, plot
- Complements: POC upward sloping (↗)
- Substitutes: POC downward sloping (↘)
Price offer curve

- **Price offer curve**: A graph of all optimal bundles for a given $m, p_2$, for all values of $p_1$
- $p_1$ varies, $m, p_2$ constant
- Plug values of $p_1$ into demand functions, plot
- Complements: POC upward sloping (↗)
- Substitutes: POC downward sloping (↘)
Demand curve

- **Demand curve**: A graph of the demand for good \( i \), for all values of \( p_i \), holding constant \( m, p_{\text{not } i} \)

- **Non-Giffin goods**: downward-sloping or flat
Perfect substitutes

\[ u(x_1, x_2) = x_1 + x_2 \]

\[ 10 = p_1 x_1 + x_2 \]

\[ x_1(p_1, 1, 10) = \begin{cases} \frac{10}{p_1} & \text{if } p_1 < 1 \\ [0, 10] & \text{if } p_1 = 1 \\ 0 & \text{if } p_1 > 1 \end{cases} \]

\[ x_2(p_1, 1, 10) = \begin{cases} 0 & \text{if } p_1 < 1 \\ 10 - x_1 & \text{if } p_1 = 1 \\ 10 & \text{if } p_1 > 1 \end{cases} \]
Cobb-Douglas

\[ u(x_1, x_2) = x_1^{0.75} x_2^{0.25} \]

\[ 40 = 2x_1 + p_2 x_2 \]

\[ x_1(2, p_2, 40) = 15, \quad x_2(2, p_2, 40) = \frac{10}{p_2} \]
Quasilinear

\[ u(x_1, x_2) = \ln(x_1) + x_2 \]
\[ 10 = p_1 x_1 + x_2 \]
\[ x_1(p_1, 1, 10) = \frac{1}{p_1}, \quad x_2(p_1, 1, 10) = 9 \]