Elasticity
Intermediate Microeconomics
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Companion to Lecture 6 / Chapter 6

1 What is elasticity?

Elasticity is an important concept in economics (micro- and macro-), and one that the textbook gives short shrift. Elasticity is a way of measuring the responsiveness of an endogenous variable to a change in parameters. Elasticity is a useful measure because its values can be understood without needing to know what units we are measuring in.

In lecture 6, we will work with 3 definitions of elasticity:

\[
\begin{align*}
\text{Income elasticity of demand } & \phi_{x,m} = \frac{dx_i}{dm} \cdot \frac{m}{x_i} \\
(Own) \text{ Price elasticity of demand } & \phi_{x,p_1} = -\frac{dx_i}{dp_i} \cdot \frac{p_i}{x_i} \\
\text{Cross price elasticity of demand } & \phi_{x,p_{-i}} = \frac{dx_i}{dp_{-i}} \cdot \frac{p_{-i}}{x_i}
\end{align*}
\]

\(i\) means an arbitrary good number. In most cases in this course, \(i\) is a stand in for "1 or 2". The notation \(p_{-i}\) refers to the price of a good other than good \(i\). Often, \(i = 1\) and \(-i = 2\).

2 Income elasticity of demand

One way to measure the effect of a $1 increase in income on demand for good \(x\) is to take the derivative \(\frac{dx}{dm}\). But the same value of \(\frac{dx}{dm}\) can have very different meanings for different goods. As an example, say \(\frac{dx}{dm} = 0.001\); ie an additional $1000 in income leads to an additional purchase of 1 unit of \(x\). This is very small if \(x\) is meals at restaurants, but very large if \(x\) is Lamborghinis.

The idea of income elasticity of demand is that if income goes up by a little bit, how much does demand go up (or down) by, and how do these two compare? Let’s illustrate this with a basic example.

Say we start at a situation with \(m = $100\) and \(p = 1\) for good \(x\). Consider a small change, \(\Delta m\), in income. \(\Delta m\) should be small relative to the current
income, so we’ll use \( m + \Delta m = \$101 \). In this case, \( \Delta m = 0.01 \times m \). We should observe a small change in demand. Say, from \( x = 10 \) to \( x + \Delta x = 10.2 \). Then, \( \Delta x = 0.02 \times x \). The percentage change in \( x \), \( \frac{\Delta x}{x} \) is twice as large as the percentage change in \( m \), \( \frac{\Delta m}{m} \). We call this ratio the arc income elasticity of demand, which we denote with \( E_{x,m} \).

\[
E_{x,m} = \frac{\Delta x}{x} \cdot \frac{m}{\Delta m} = 0.02/0.01 = 2
\]

Say we look at a smaller percentage change in \( m \), say \( \Delta m = 0.001 \times m \), and find that \( \Delta x = 0.0027 \times x \). Then we get a new measure of arc elasticity.

\[
E_{x,m} = 0.0027/0.001 = 2.7
\]

The exact value of arc elasticity depends on our choice of \( \Delta m \), and requires computing the quantity demanded every time we consider arc elasticity at a different set of \( p_1, p_2, m \). This is somewhat arbitrary, and a bit of a hassle to do repeatedly. It is more useful to use calculus, which will give us a function into which we can plug in values \( p_1, p_2, m \), and spares us the consideration of what is a sufficiently small value for \( \Delta m \).

To see how this is done, rearrange terms in the formula for arc elasticity:

\[
E_{x,m} = \frac{\Delta x}{\Delta m} \cdot \frac{m}{x}
\]

Make \( \Delta m \) arbitrarily small.

\[
\epsilon_{x,m} = \lim_{\Delta m \to 0} \frac{\Delta x(\Delta m)}{\Delta m} \cdot \frac{m}{x}
= \lim_{\Delta m \to 0} \frac{x(m + \Delta m) - x(m)}{\Delta m} \cdot \frac{m}{x}
= \frac{dx}{dm} \cdot \frac{m}{x}
\]

This is less work. Say \( \frac{dx}{dm} = 0.3 \) for our example above, then \( \epsilon_{x,m} = 0.3 \times \frac{100}{10} = 3 \).

So, in the neighborhood of \( m = \$100 \), the optimal value of \( x \) grows three times as fast as income, as \( m \) increases. This implies that as \( m \) increases, the proportion of income that is spent on \( x \) increases, at least for values of \( m \) near \$100.

### 2.1 Luxury goods

A **luxury good** is any good, like \( x \) in the example above, for which the income elasticity of demand is greater than 1. These are goods that make up a larger share of expenditure as income goes up.

A **necessary good** is any good, for which the income elasticity of demand is less than 1. These are goods that make up a smaller share of expenditure as income goes up.

Note that a good can be a luxury for some values of \((p_1, p_2, m)\), and a necessary good for other values of \((p_1, p_2, m)\).
3 (Own) Price elasticity of demand

We may also want to look at comparative statics in terms of prices. What happens to demand for a good as its price increases? Just like with income, we could take the derivative $\frac{dx_i}{dp_i}$. Not surprisingly, $\frac{dx_i}{dp_i} < 0$, for virtually every good. However, what if we wanted to know about expenditure on good $i$? Expenditure on good $i$ is

$$p_i x_i(p_1, p_{-i}, m)$$

Take the derivative wrt $p_i$:

$$\frac{d}{dp_i} p_i x_i(p_1, p_{-i}, m) = p_i \frac{dx_i}{dp_i} + x_i$$

Expenditure on $x_i$ is increasing in $p_i$ when

$$p_i \frac{dx_i}{dp_i} + x_i > 0$$

$$p_i \frac{dx_i}{dp_i} > -x_i$$

$$- \frac{dx_i}{dp_i} > 1$$

$$\epsilon_{x_i,p_i} > 1$$

This last step takes our formula for own price elasticity of demand, and substitutes it in. Expenditure on good $i$ increases in response to ↑ $p_i$ if $\epsilon_{x_i,p_i} > 1$.

3.1 Elastic demand

A good has **elastic** demand if its own price elasticity of demand, $\epsilon_{x_i,p_i}$ is greater than one.

A good has **inelastic** demand if its own price elasticity of demand, $\epsilon_{x_i,p_i}$ is less than one.

A good has **unit elastic** demand if its own price elasticity of demand, $\epsilon_{x_i,p_i}$ is exactly one.

It can help to think of own price elasticity as how much the quantity demanded can stretch in response to a change in price.

4 Cross price elasticity of demand

Finally, we may be curious what happens to demand for a good (say, $x_1$) as the price (say, $p_2$) of another good increases? Logically, if expenditure on $x_2$ increases, that extra cost must come out of expenditure on another good. So, in a 2-good market, if $\epsilon_{x_2,p_2} < 1$, $\frac{dx_1}{dp_2} < 0$. We are primarily interested in the sign of this effect, so this first derivative will suffice. However, if we wanted to
make comparisons of the magnitude of these effects across goods, we will need to use cross price elasticity:

$$\epsilon_{x_1, p_2} = \frac{dx_1}{dp_2} \cdot \frac{p_2}{x_1}$$

### 4.1 Complements/Substitutes

Two goods are **substitutes** if their cross price elasticity of demand, $\epsilon_{x_i, p_{-i}}$, is positive.

Two goods are **complements** if their cross price elasticity of demand, $\epsilon_{x_i, p_{-i}}$, is negative.

Suppose $p_2$ increases. If your response is to spend more on good 2, and less on good 1, you are trying to keep a balance between the two goods, as if they go together. That’s why they’re called complements. If, instead, your response is to buy less of good 2, and more of good 1, good 1 is substituting for good 2.